

# 10th Class 2020

Math (Science)	Group-I	PAPER-II
Time: 2.10 Hours	(Subjective Type)	Max. Marks: 60

(Part-I)

2. Write short answers to any SIX (6) questions: (12)

(i) Define exponential equation and give an example.

**Ans** In an equation, if variable occurs in exponent, then it is called exponential equation.

For example,  $5^{1+x} + 5^{1-x} = 26$ .

(ii) Solve:  $(2x - \frac{1}{2})^2 = \frac{9}{4}$ .

**Ans** Given:  $(2x - \frac{1}{2})^2 = \frac{9}{4}$

By taking under root both sides, we get

$$\sqrt{(2x - \frac{1}{2})^2} = \sqrt{\frac{9}{4}}$$

$$2x - \frac{1}{2} = \pm \frac{3}{2}$$

$$2x - \frac{1}{2} = \frac{3}{2} \quad ; \quad 2x - \frac{1}{2} = -\frac{3}{2}$$

$$2x = \frac{3}{2} + \frac{1}{2} \quad ;$$

$$2x = -\frac{3}{2} + \frac{1}{2}$$

$$= \frac{3+1}{2} \quad ;$$

$$= \frac{-3+1}{2}$$

$$= \frac{4}{2} \quad ;$$

$$= -\frac{2}{2}$$

$$2x = 2 \quad ;$$

$$2x = -1$$

$$\boxed{x = 1} \quad ;$$

$$\boxed{x = -\frac{1}{2}}$$

The solution set will be:  $\left\{1, -\frac{1}{2}\right\}$ .

(iii) Solve the given equation using quadratic formula:

$$2 - x^2 = 7x$$

**Ans**

Given quadratic equation:

$$2 - x^2 = 7x$$

By arranging the above equation as standard form:

$$0 = 7x + x^2 - 2$$

$$\Rightarrow x^2 + 7x - 2 = 0$$

From above equation,

$$a = 1, b = 7, c = -2$$

The Quadratic Formula:

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-7 \pm \sqrt{(7)^2 - 4(1)(-2)}}{2(1)} \\ &= \frac{-7 \pm \sqrt{49 + 8}}{2} \\ &= \frac{-7 \pm \sqrt{57}}{2} \end{aligned}$$

The solution set will be:

$$\left\{ \frac{-7 \pm \sqrt{57}}{2} \right\}$$

(iv) Evaluate:  $(1 - \omega + \omega^2)^6$

**Ans**

Given:  $(1 - \omega + \omega^2)^6$

The above expression can be written as:

$$(1 + \omega^2 - \omega)^6$$

$$\text{So, } (1 + \omega^2 - \omega)^6 = (-\omega - \omega)^6 \quad (\because 1 + \omega^2 = -\omega)$$

$$= (-2\omega)^6$$

$$= (-2)^6 \omega^6$$

$$= 64 (\omega^3)^2$$

$$= 64 (1)^2$$

$$= 64$$

$$(\because \omega^3 = 1)$$



- (v) Using synthetic division, show that  $x - 2$  is a factor of  $x^3 + x^2 - 7x + 2$ .

**Ans**  $P(x) = x^3 + x^2 - 7x + 2$   
And  $x - a = x - 2$   
So,  $a = 2$   
By synthetic division,

$$\begin{array}{r|rrrr} & 1 & 1 & -7 & 2 \\ 2 & \downarrow & 2 & 6 & -2 \\ \hline & 1 & 3 & -1 & 0 \end{array}$$

- (vi) Write the quadratic equation having roots: 0, -3.

**Ans** Sum of the roots =  $S = 0 + (-3)$   
 $= 0 - 3$   
 $= -3$

Product of the roots =  $P = 0(-3)$   
 $= 0$

The standard form of quadratic equation, having roots, is:

$$x^2 - Sx + P = 0$$

By putting the values of  $S$  and  $P$ , we get

$$x^2 - (-3)x + 0 = 0$$

$$x^2 + 3x = 0 \quad (\text{Required Equation}).$$

- (vii) Define proportion.

**Ans** A proportion is a statement, which is expressed as an equivalence of two ratios. If two ratios  $a : b$  and  $c : d$  are equal, then we can write  $a : b :: c : d$ ; where quantities  $a, d$  are called extremes, while  $b, c$  are called means.

- (viii) If  $w \propto \frac{1}{v^2}$  and  $w = 2$  when  $v = 3$ , then find  $w$ .

**Ans** Given,

$$w \propto \frac{1}{v^2}$$

$$\Rightarrow w = \frac{k}{v^2} \quad (i)$$

By putting  $w = 2$  and  $v = 3$  in equation (i), we get

$$2 = \frac{k}{(3)^2}$$

$$2 = \frac{k}{9}$$

$$\Rightarrow \boxed{k = 18}$$

Now putting  $k = 18$  in equation (i),

$$w = \frac{18}{(3)^2}$$

$$w = \frac{18}{9}$$

$$\boxed{w = 2}$$

(ix) Find a third proportional to:  $a^2 - b^2, a - b$ .

**Ans** Let  $x =$  third proportional

$$a^2 - b^2 : (a - b) :: (a - b) : x$$

Product of Extremes = Product of Means

$$x(a^2 - b^2) = (a - b)(a - b)$$

$$x = \frac{(a - b)(a - b)}{(a^2 - b^2)}$$

$$x = \frac{(a - b)(a - b)}{(a + b)(a - b)}$$

$$x = \frac{a - b}{a + b}$$

Thus, the third proportional is  $\frac{a - b}{a + b}$ .

**3. Write short answers to any SIX (6) questions: (12)**

(i) Define improper fraction with an example.

**Ans** A rational fraction  $\frac{N(x)}{D(x)}$ , with  $D(x) \neq 0$  is called an improper fraction, if degree of the polynomial  $N(x)$  is greater or equal to the degree of the polynomial  $D(x)$ .

For example;



$$\frac{5x}{x+2}, \frac{3x^2+2}{x^2+7x+12}, \frac{6x^4}{x^3+1}$$

are improper fractions.

(ii) Resolve  $\frac{5x+4}{(x-4)(x+2)}$  into partial fraction.

**Ans** Let  $\frac{5x+4}{(x-4)(x+2)} = \frac{A}{x-4} + \frac{B}{x+2}$  (i)

Multiplying throughout by  $(x-4)(x+2)$ , we get

$$5x+4 = A(x+2) + B(x-4) \quad (ii)$$

Equation (ii) is an identity, which holds good for all values of  $x$  and hence for

$$x=4 \text{ and } x=-2$$

Put  $x-4=0$  i.e.,  $x=4$  (factor corresponding to  $A$ ) on both sides of the equation (ii), we get

$$5(4)+4 = A(4+2)$$

$$20+4 = A(6)$$

$$\frac{24}{6} = A$$

$$\Rightarrow \boxed{A=4}$$

Put  $x+2=0$  i.e.,  $x=-2$  (factor corresponding to  $B$ ), we get

$$5(-2)+4 = B(-2-4)$$

$$-10+4 = B(-6)$$

$$\Rightarrow -6B = -6$$

$$\Rightarrow \boxed{B=1}$$

Thus, required partial fractions are  $\frac{4}{x-4} + \frac{1}{x+2}$ .

$$\text{Hence, } \frac{5x+4}{(x-4)(x+2)} = \frac{4}{x-4} + \frac{1}{x+2}$$

(iii) If  $X = \phi$ ,  $Y = Z^+$ , then find  $X \cap Y$ .

**Ans** Given,

$$\begin{aligned} X \cap Y &= \phi \cap \{0, 1, 2, 3, \dots\} \\ &= \phi \end{aligned}$$

(iv) Find  $a$  and  $b$ , if  $(2a + 5, 3) = (7, b - 4)$ .

**Ans** By comparing the values, we get

$$2a + 5 = 7 \quad ; \quad 3 = b - 4$$

$$2a = 7 - 5 \quad ; \quad 3 + 4 = b$$

$$2a = 2 \quad ; \quad 7 = b$$

$$a = \frac{2}{2} \quad ; \Rightarrow \boxed{b = 7}$$

$$\boxed{a = 1}$$

Thus,  $\{a = 1, b = 7\}$ .

(v) If set  $M$  has 5 elements, then find the numbers of binary relations in  $M$ .

**Ans** Number of elements in  $M = 5$

$$\begin{aligned} \text{Number of binary relations in } M &= 2^{5 \times 5} \\ &= 2^{25} \end{aligned}$$

(vi) Define a bijective function.

**Ans** A function  $f : A \rightarrow B$  is called bijective function 'f' is one-one and onto. For example, if  $A = \{1, 2, 3, 4\}$  and  $B = \{2, 3, 4, 5\}$ .

(vii) The marks of seven students in Mathematics are as follows, calculate the arithmetic mean:

Student No.	1	2	3	4	5	6	7
Marks	45	60	74	58	65	63	49

**Ans**

Student No.	Marks (X)
1	45
2	60
3	74
4	58
5	65
6	63
7	49
	414

For arithmetic Mean ( $\bar{X}$ ):

$$\begin{aligned} \bar{X} &= \frac{\Sigma X}{n} \\ &= \frac{414}{7} \end{aligned}$$

$$\boxed{\bar{X} = 59.14}$$



- (viii) Find the modal size of shoes for the following data:  
4, 4, 5, 5, 6, 6, 6, 7, 7, 5, 7, 5, 8, 8, 8, 6, 5, 6, 5, 7

**Ans** As the number '5' is repeated most of the times in the given data, so the modal size of shoes is 5.

- (ix) Define median and write its formula.

**Ans** Median:

Median is the middle most observation in an arranged data set. It divides the data set into two equal parts.  $\tilde{X}$  is used to represent median. We can determine median by using the following formulas:

**Case-1:**

When number of observation is odd:

$$\tilde{X} = \text{size of } \left(\frac{n+1}{2}\right)\text{th value.}$$

**Case-2:**

When number of observations is even:

$$\tilde{X} = \frac{1}{2} \left[ \text{size of } \left(\frac{n}{2}\right)\text{th} + \left(\frac{n+1}{2}\right)\text{th value} \right].$$

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4. Write short answers to any SIX (6) questions: (12)

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- (i) Convert  $\frac{3\pi}{4}$  radians to degrees.

**Ans**

$$\begin{aligned}\frac{3\pi}{4} &= \frac{3\pi}{4} \text{ Radian} \\ &= \frac{3\pi}{4} \times 1 \text{ Radian} \\ &= \frac{3\pi}{4} \times \frac{180^\circ}{\pi} \\ &= 135^\circ\end{aligned}$$

- (ii) Find 'r', when  $l = 52$  cm,  $\theta = 45^\circ$ .

**Ans** As we know that,

$$\theta = 45 \times \frac{\pi}{180} = \frac{\pi}{4}$$

$$\begin{aligned}
 r &= \frac{l}{\theta} \\
 &= l \div \theta \\
 &= 52 \div \frac{\pi}{4} \\
 &= 52 \times \frac{4}{\pi} \\
 &= 52 \times 1.273
 \end{aligned}$$

$$r = 66.20$$

(iii) In a  $\triangle ABC$ ,  $a = 17$  cm,  $b = 15$  cm and  $c = 8$  cm, find  $m\angle A$ .

**Ans** By Pythagora's Theorem:

$$\begin{aligned}
 a^2 &= c^2 + b^2 \\
 (17)^2 &= (8)^2 + (15)^2 \\
 289 &= 64 + 225 \\
 289 &= 289
 \end{aligned}$$

(iv) Define diameter of a circle.

**Ans** A chord which passes through the centre of the circle is called diameter of a circle.

(v) Define secant of a circle.

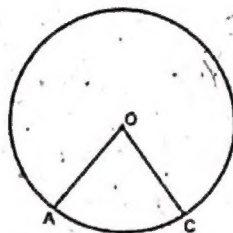
**Ans** A secant is a straight line which cuts the circumference of a circle in two distinct points.

(vi) Define circumference of the circle.

**Ans** The length of the boundary of the circle is called the circumference. It is calculated by  $2\pi r$ .

(vii) Define central angle of a circle.

**Ans**  $\angle AOC$  is the central angle of the circle whose vertex is at the centre  $O$  and its arms meet at the end points of arc  $\widehat{AC}$ .





(viii) Define circum circle.

**Ans** The circle passing through the vertices of triangle ABC is known as circum circle. Its radius as circum radius and centre as circum centre.

(ix) The length of each side of a regular octagon is 3 cm. Measure its perimeter.

**Ans** Length of side = 3 cm  
Number of sides of an octagon = 8  
Perimeter = Length  $\times$  sides  
 $= 3 \times 8$   
 $= 24 \text{ cm}$

(Part-II)

**NOTE:** Attempt THREE (3) questions in all. But question No. 9 is Compulsory.

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**Q.5.(a) Solve the equation:  $2x^4 = 9x^2 - 4$ . (4)**

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**Ans** For Answer see Paper 2019 (Group-I), Q.5.(a).

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**(b) Solve by using synthetic division if  $-1$  is the root of the equation  $4x^3 - x^2 - 11x - 6 = 0$ . (4)**

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**Ans** Since  $-1$  is the root of the equation:

$$4x^3 - x^2 - 11x - 6 = 0$$

Then by synthetic division, we get

$-1$	$4$	$-1$	$-11$	$-6$
	$\downarrow$	$-4$	$5$	$6$
	$4$	$-5$	$-6$	$0$

The depressed equation is:

$$4x^2 - 5x - 6 = 0$$

$$4x^2 - 8x + 3x - 6 = 0$$

$$4x(x - 2) + 3(x - 2) = 0$$

$$(4x + 3)(x - 2) = 0$$

$$4x + 3 = 0 \quad ; \quad x - 2 = 0$$

$$4x = -3 \quad ; \quad x = 2$$

$$x = \frac{-3}{4}$$

Hence,  $\frac{-3}{4}$ , 2 and -1 are the roots of the given equation.

Q.6.(a) If  $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$  ( $a, b, c, d, e, f \neq 0$ ), then show that

$$\frac{ac + ce + ea}{bd + df + fb} = \left[ \frac{ace}{bdf} \right]^{2/3} \quad (4)$$

**Ans** Let  $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = k$

$$\Rightarrow \frac{a}{b} = k \quad \text{and} \quad \frac{c}{d} = k \quad \text{and} \quad \frac{e}{f} = k$$

$$\Rightarrow a = bk \quad \text{and} \quad c = dk \quad \text{and} \quad e = fk$$

$$\begin{aligned} \text{L.H.S} &= \frac{ac + ce + ea}{bd + df + fb} \\ &= \frac{(bk)(dk) + (dk)(fk) + (fk)(bk)}{bd + df + fb} \\ &= \frac{bd k^2 + df k^2 + fb k^2}{bd + df + fb} \\ &= \frac{k^2 (bd + df + fb)}{bd + df + fb} \\ &= k^2 \end{aligned} \quad (i)$$

$$\begin{aligned} \text{R.H.S} &= \left[ \frac{ace}{bdf} \right]^{2/3} \\ &= \left[ \frac{(bk)(dk)(fk)}{bdf} \right]^{2/3} \\ &= \left[ \frac{bdf k^3}{bdf} \right]^{2/3} \\ &= (k^3)^{2/3} \\ &= k^2 \end{aligned} \quad (ii)$$

From (i) and (ii), we have

$$\text{L.H.S} = \text{R.H.S}$$

$$\text{Hence, } \frac{ac + ce + ea}{bd + df + fb} = \left[ \frac{ace}{bdf} \right]^{2/3}$$



(b) Resolve into partial fractions:  $\frac{3x+7}{(x^2+1)(x+3)}$ . (4)

**Ans**  $\frac{3x+7}{(x^2+1)(x+3)} = \frac{Ax+B}{x^2+1} + \frac{C}{x+3}$

Multiplying throughout by  $(x^2+1)(x+3)$ , we get

$$3x+7 = (Ax+B)(x+3) + C(x^2+1) \quad (i)$$

$$3x+7 = Ax^2 + 3Ax + Bx + 3B + Cx^2 + C$$

$$3x+7 = Ax^2 + Cx^2 + 3Ax + Bx + 3B + C \quad (ii)$$

To find C:

We put  $x+3=0 \Rightarrow x=-3$  in equation (i), we get

$$3(-3)+7 = (A(-3)+B)(-3+3) + C((-3)^2+1)$$

$$-9+7 = (-3A+B)(0) + C(9+1)$$

$$-2 = 10C$$

$$10C = -2$$

Dividing throughout by '10', we get

$$C = \frac{-1}{5}$$

To find A and B:

Equating coefficient of  $x^2$  and constants on both sides of eq (ii), we get

$$A + C = 0$$

$$A + \left(\frac{-1}{5}\right) = 0$$

$$A = \frac{1}{5}$$

$$\text{And } 3B + C = 7$$

$$3B + \left(\frac{-1}{5}\right) = 7$$

$$3B = 7 + \frac{1}{5}$$

$$3B = \frac{35+1}{5}$$

$$3B = \frac{36}{5}$$

$$B = \frac{36}{5} \times \frac{1}{3}$$

$$B = \frac{12}{5}$$

Thus, required partial fractions are

$$\begin{aligned} \frac{\frac{1}{5}x + \frac{12}{5}}{x^2 + 1} + \frac{\frac{-1}{5}}{x + 3} \\ = \frac{\frac{x + 12}{5}}{x^2 + 1} - \frac{\frac{1}{5}}{x + 3} \\ = \frac{x + 12}{5(x^2 + 1)} - \frac{1}{5(x + 3)} \end{aligned}$$

Hence,  $\frac{3x - 7}{(x^2 + 1)(x + 3)} = \frac{x + 12}{5(x^2 + 1)} - \frac{1}{5(x + 3)}$

Q.7.(a) If  $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ ,  $A = \{1, 3, 5, 7, 9\}$  and  $B = \{2, 3, 5, 7\}$ , then verify  $(A \cap B)' = A' \cup B'$ . (4)

**Ans** For Answer see Paper 2019 (Group-I), Q.7.(a).

(b) The marks of six students in Mathematics are given, determine variance. (4)

Student	1	2	3	4	5	6
Marks	60	70	30	90	80	42

**Ans** For Answer see Paper 2017 (Group-II), Q.7.(b).

Q.8.(a) Verify the identity: (4)

$$\cos^4 \theta - \sin^4 \theta = \cos^2 \theta - \sin^2 \theta$$

**Ans**

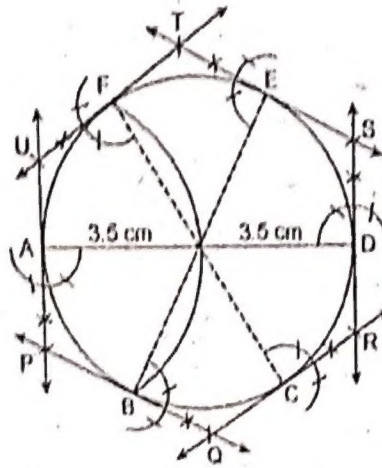
$$\begin{aligned} \text{L.H.S} &= \cos^4 \theta - \sin^4 \theta \\ &= (\cos^2 \theta)^2 - (\sin^2 \theta)^2 \\ &= (\cos^2 \theta + \sin^2 \theta)(\cos^2 \theta - \sin^2 \theta) \\ &= 1(\cos^2 \theta - \sin^2 \theta) \\ &= \cos^2 \theta - \sin^2 \theta \\ &= \text{R.H.S} \end{aligned}$$

Proved.



(b) About a circle of radius 3.5 cm, describe a regular hexagon.

**Ans**

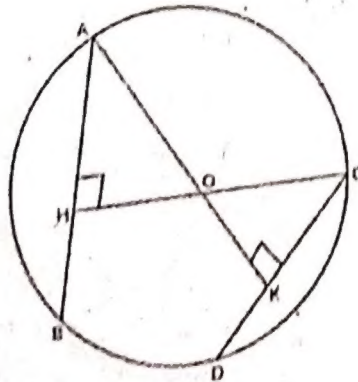


**Steps of Construction:**

1. Draw a diameter  $\overline{AD} = 7$  cm.
2. From point A draw an arc of radius  $\overline{AO} = 3.5$  cm (the radius of the circle), which cuts the circle at points B and F.
3. Join B with O and extend it to meet the circle at E.
4. Join F with O and extend it to meet the circle at C.
5. Draw tangents to the circle at points A, B, C, D, E and F intersecting one another at points P, Q, R, S, T and U, respectively.
6. Thus PQRSTU is the circumscribed regular hexagon.

**Q.9. Prove that two chords of a circle which are equidistant from the centre are congruent. (8)**

**Ans**



Given:

$\overline{AB}$  and  $\overline{CD}$  are two chords of a circle with center at O.  
 $\overline{OH} \perp \overline{AB}$  and  $\overline{OK} \perp \overline{CD}$ , so that  $m\overline{OH} = m\overline{OK}$ .

To Prove:

$$m\overline{AB} = m\overline{CD}$$

Construction:

Join A and C with O, so that we can form  $\angle$ rt  $\Delta^s$  OAH and OCK.

Proof:

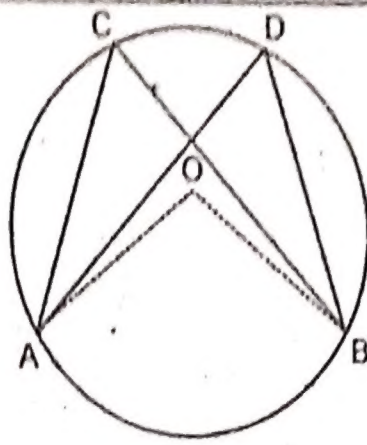
Statements	Reasons
In $\angle$ rt $\Delta^s$ OAH $\leftrightarrow$ OCK,	
$\therefore$ hyp. $\overline{OA} =$ hyp. $\overline{OC}$	Radii of the same circle
$m\overline{OH} = m\overline{OK}$	Given
$\therefore \Delta$ OAH $\cong$ $\Delta$ OCK	H.S postulate
So,	
$m\overline{AH} = m\overline{CK}$ (i)	Corresponding sides of congruent triangles
But	
$m\overline{AH} = \frac{1}{2} m\overline{AB}$ (ii)	$\overline{OH} \perp$ chord $\overline{AB}$ (Given)
Similarly,	
$m\overline{CK} = \frac{1}{2} m\overline{CD}$ (iii)	$\overline{OK} \perp$ chord $\overline{CD}$ Given
Since $m\overline{AH} = m\overline{CK}$	Already proved in (i)
$\therefore \frac{1}{2} m\overline{AB} = \frac{1}{2} m\overline{CD}$	Using (ii) and (iii)
or $m\overline{AB} = m\overline{CD}$	

OR

Prove that any two angles in the same segment of a circle are equal.

Ans





**Given:**

$m\angle ACB = m\angle ADB$  are the circumangles in the same segment of a circle with centre O.

**To Prove:**

$$m\angle ACB = m\angle ADB$$

**Construction:**

Join O with A and O with B.

So that  $\angle AOB$  is the central angle.

**Proof:**

Statements	Reasons
Standing on the same arc AB of a circle.	
$\angle AOB$ is the central angle whereas $\angle ACB$ and $\angle ADB$ are circumangles	Construction
$\therefore m\angle AOB = 2m\angle ACB$ (i)	Given
	By theorem I (External angle is the sum internal opposite angle).
and $m\angle AOB = 2m\angle ADB$ (ii)	By theorem I
$\Rightarrow 2m\angle ACB = 2m\angle ADB$	Using (i) and (ii)
Hence,	
$m\angle ACB = m\angle ADB$	